

Chapter

# 2

# Algebra techniques

## Syllabus Content

### A – Basic Mathematics – 10%

- Basic algebraic techniques and the solution of equations.

## 2.1 What is algebra?

In order to extend the usefulness of mathematical techniques we introduce letters or symbols to represent numbers. For example we may introduce the letter “ $i$ ” to represent the interest rate. The choice of letter is down to the user’s preference. Very often “ $x$ ” and “ $y$ ” are used to represent numbers in algebra. Algebra is the manipulation of these symbols.

### Arithmetic of symbols

#### Addition

- Letters  $x$  and  $y$  stand for 2 numbers then their sum is  $x + y$  or  $y + x$ .

#### Subtraction

- $x - y$  means the number  $y$  is subtracted from the number  $x$ .

#### Multiplication

- $x \times y$  means  $x$  multiplied by  $y$ . the multiplied sign is often replaced with “ $\cdot$ ” Or removed totally. Therefore can be written as  $x \cdot y$  or  $xy$

#### Division

- $x/y$  means  $x$  divided by  $y$ .

### Example 2.1 Substitution

(a) Find the value of  $b^5$  when  $b = 4$ .

(b) Find the value of  $d + 7e - 4f$  when  $d = 5$ ,  $e = 4$  and  $f = 2$

(c) If  $g = 3$  find the value of  $8g^3$  and  $(8g)^3$

(d) Find the value of  $4x^3/2 + 7x$  when  $x = 4$

## Simplifying expressions

Simplifying an algebraic expression means making it neater and usually shorter by combining terms where possible. This involves collecting like terms into groups (e.g. all the “x” or all the “y”) and then combining the like terms in each group (adding, subtracting, multiplying or dividing them).

### Example 2.2 Simplifying expressions

(a) Simplify:  $5x + 2x - 8x =$

(b) Simplify:  $2t + 5i =$


(c) Simplify:  $s + 13s + s^2 =$

(d) Simplify:  $wr + w^2 - 4r^2 + 8wr + 9w^2 =$

## Multiplying brackets from expressions of the form $(a + b)(c + d)$


For these expressions we use the “FOIL” technique.

1. “**F**irst” multiply “a” with “c”



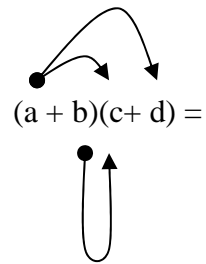
$(a + b)(c + d) =$

2. “**O**ver” multiply “a” with “d”

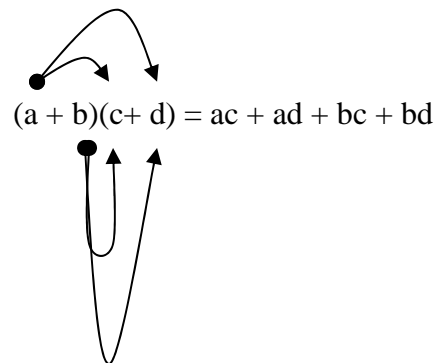


$(a + b)(c + d) =$

3. **“Inside”** multiply “b” with “c”



4. **“Last”** multiply “b” with “d”



### Example 2.3 Expansion of brackets

Expand the following:

(a)  $(3 + x)(2 + y) =$

(b)  $(x + 6)(x - 3) =$

(c)  $(1 - d)(2 - d) =$

(d)  $(-p - 2)(2p - 1) =$

### Example 2.3 Expansion of brackets (continued)

Expand the following:

(e)  $5(x + 1)(x - 1) =$

### Transposing equations

In the equation  $a = b + c/d$  we say that  $a$  is the subject of the equation. The subject appears by itself on one side, usually the left, of the equation, and nowhere else. If we are asked to transpose the equation for  $b$ , then we must rearrange the equation so that  $b$  becomes the subject. The rules for transposing are as follows, but the key rule is *“whatever you do to one side do to the other side”*

- Add the same quantity to both sides
- Subtract the same quantity from both sides
- Multiply or divide both sides by the same quantity
- Perform operations on both sides, such as “square both sides”, “square root both sides” etc...

For example:

- Make “ $b$ ” the subject in the equation:
  - $a = b + c/d$
  - $a - c/d = b + c/d - c/d$  (subtract both sides by “ $c/d$ ”, to get “ $b$ ” on its own)
  - $a - c/d = b$
  - $b = a - c/d$
- Make “ $c$ ” the subject in the equation:
  - $a = b + c/d$
  - $a - b = b + c/d - b$  (subtract both sides by “ $b$ ”, to get “ $c/d$ ” on its own)
  - $a - b = c/d$
  - $(a - b) \times d = (c/d) \times d$  (multiply both sides by “ $d$ ”, to get “ $c$ ” on its own)
  - $d(a - b) = c$

- Make “d” the subject in the equation:
  - $a = b + c/d$
  - $a - b = b + c/d - b$  (subtract both sides by “b”, to get “c/d” on its own)
  - $a - b = c/d$
  - $(a - b) / c = (c/d) / c$  (divide both sides by “c”, to get “d” on its own)
  - $(a-b)/c = 1/d$
  - $1 / ((a-b)/c) = 1(1/d)$  (reciprocate or divide both sides by “1”)
  - $c/(a-b) = d$

#### **Example 2.4 Transposing equations**

Re-arrange the following to make “g” the subject:

(a)  $y = 7g$

(b)  $y = 10g - 4y$

(c)  $y = 2/g$

(d)  $y = 5/(9g + 4)$

(e) Rearrange  $y = ax + b$  for  $x$

### Example 2.4 Transposing equations (continued)

Solve the following equations for  $y$ :

(f)  $10(y + 4) = 24$

(g)  $4(y - 3) = 6(y - 9)$

(h)  $8(2 - 3y) = 8(1 - 4y)$

(i)  $(2y + 4)/5 = (y - 3)/2$

(j)  $(y - 7)/8 = (3y + 1)/5$

(k)  $1/(y + 1) = 1/(3y + 2)$

(l)  $3/(y + 1) = 2/(4y + 1)$

## **Solutions to lecture examples**



## Chapter 2

### Example 2.1 Substitution

(a) Find the value of  $b^5$  when  $b = 4$ .

$$b^5 = b \times b \times b \times b \times b$$

If  $b = 4$

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$$

(b) Find the value of  $d + 7e - 4f$  when  $d = 5$ ,  $e = 4$  and  $f = 2$

If we substitute all the numbers for their respective letters:

$$5 + (7 \times 4) - (4 \times 2) = 5 + 28 - 8 = 25$$

(c) If  $g = 3$  find the value of  $8g^3$  and  $(8g)^3$

$$8g^3 = 8(3)^3 = 8 \times 27 = 216$$

$$(8g)^3 = (8 \times 3)^3 = 24^3 = 13824$$

(d) Find the value of  $4x^3/2 + 7x$  when  $x = 4$

Therefore,

$$4(4)^3/2 + (7 \times 4) = 4(4 \times 4 \times 4)/2 + 28 = (4 \times 64)/2 + 28 = 256/2 + 28 = 128 + 28 = 156$$

### Example 2.2 Simplifying expressions

(a) Simplify:  $5x + 2x - 8x = -x$

(b) Simplify:  $2t + 5i =$  can't simplify any further because letters cannot be grouped.

(c) Simplify:  $s + 13s + s^2 = s^2 + 14s = s(s + 14)$

(d) Simplify: 
$$\begin{aligned} & wr + w^2 - 4r^2 + 8wr + 9w^2 \\ &= -4r^2 + (9w^2 + w^2) + (8wr + wr) \\ &= -4r^2 + 10w^2 + 9wr \end{aligned}$$

### Example 2.3 Expansion of brackets

Expand the following:

$$\begin{aligned} \text{(a)} \quad (3 + x)(2 + y) &= \\ (3 \times 2) + 3y + 2x + xy &= \\ 6 + 3y + 2x + xy & \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (x + 6)(x - 3) &= \\ x^2 + -3x + 6x + (6 \times -3) &= \\ x^2 + -3x + 6x - 18 &= \\ x^2 + 3x - 18 & \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (1 - d)(2 - d) &= \\ (1 \times 2) + (1 \times -d) + (-2 \times d) + (-d \times -d) &= \\ 2 - d - 2d + d^2 &= \\ 2 - 3d + d^2 & \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad (-p - 2)(2p - 1) &= \\ (-p \times 2p) + (-p \times -1) + (-2 \times 2p) + (-2 \times -1) &= \\ -2p^2 + p - 4p + 2 &= \\ -2p^2 - 3p + 2 & \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 5(x + 1)(x - 1) &= \\ 5((x \times x) + (-1 \times x) + (1 \times x) + (1 \times -1)) &= \\ 5(x^2 + -x + x + -1) &= \\ 5(x^2 + -1) &= \\ 5(x^2 - 1) &= \\ 5x^2 - 5 & \end{aligned}$$

### Example 2.4 Transposing equations

Re-arrange the following to make “g” the subject:

$$\begin{aligned} \text{(a)} \quad y &= 7g \\ y/7 &= 7g/7 \text{ (divide both sides by 7, to leave 'g' on its own)} \\ y/7 &= g \\ g &= y/7 \end{aligned}$$

(b)  $y = 10g - 4y$   
 $y + 4y = 10g - 4y + 4y$  (add "4y" to both sides, to leave '10g' on its own)  
 $5y = 10g$   
 $5y/10 = 10g/10$  (divide both sides by 10, to leave 'g' on its own)  
 $y/2 = g$   
 $g = y/2$

(c)  $y = 2/g$   
 $1/y = g/2$  (reciprocate both sides)  
 $(1/y) \times 2 = (g/2) \times 2$  (multiply both sides by 2)  
 $2/y = g$   
 $g = 2/y$

(d)  $y = 5/(9g + 4)$   
 $1/y = (9g + 4)/5$  (reciprocate both sides)  
 $(1/y) \times 5 = ((9g + 4)/5) \times 5$  (multiply both sides by 5)  
 $5/y = 9g + 4$   
 $5/y - 4 = 9g + 4 - 4$  (subtract both sides by 4)  
 $5/y - 4 = 9g$   
 $(5/y - 4)/9 = 9g/9$  (divide both sides by 9)  
 $(5/y - 4)/9 = g$   
 $g = (5/y - 4)/9$

(e) Rearrange  $y = ax + b$  for  $x$   
 $y - b = ax + b - b$  (subtract both sides by "b")  
 $(y - b)/a = x$  (divide both sides with "a")  
 $x = (y - b)/a$

Solve the following equations for y:

(f)  $10(y + 4) = 24$   
 $y + 4 = 24/10$  (divide both sides by 10)  
 $y = 2.4 - 4$  (subtract both sides by 4)  
 $y = -1.6$

(g)  $4(y - 3) = 6(y - 9)$   
 $4y - 12 = 6y - 48$  (multiply out brackets on both sides)  
 $4y - 6y - 12 = 6y - 6y - 48$  (subtract "6y" from both sides)  
 $-2y - 12 = -48$   
 $-2y - 12 + 12 = -48 + 12$  (add both sides by 12)  
 $-2y = -36$   
 $-2y/-2 = -36/-2$  (divide both sides by -2)  
 $y = 18$

(h)  $8(2 - 3y) = 8(1 - 4y)$

$16 - 24y = 8 - 32y$  (multiply out brackets on both sides)

$16 - 24y + 32y = 8 - 32y + 32y$  (add "32y" on both sides)

$16 + 8y = 8$

$16 - 16 + 8y = 8 - 16$  (subtract both sides by 16)

$8y = -8$  (divide both sides by 8)

$8y/8 = -8/8$

$y = -1$

(i)  $(2y + 4)/5 = (y - 3)/2$

Eliminate fractions, multiply both sides by 5 and 2

$2 \times 5 \times (2y + 4)/5 = 2 \times 5 \times (y - 3)/2$

$2(2y + 4) = 5(y - 3)$

$4y + 8 = 5y - 15$  (multiply out brackets on both sides)

$4y - 5y + 8 = 5y - 5y - 15$  (subtract "5y" from both sides)

$-y + 8 = -15$

$-y + 8 - 8 = -15 - 8$  (subtract both sides by 8)

$-y = -23$  (multiply both sides by -1)

$y = 23$

(j)  $(y - 7)/8 = (3y + 1)/5$

Eliminate fractions, multiply both sides by 8 and 5

$5 \times 8 \times (y - 7)/8 = 5 \times 8 \times (3y + 1)/5$

$5(y - 7) = 8(3y + 1)$

$5y - 35 = 24y + 8$  (multiply out brackets on both sides)

$5y - 5y - 35 = 24y - 5y + 8$  (subtract "5y" from both sides)

$-35 = 19y + 8$

$-35 - 8 = 19y + 8 - 8$  (subtract both sides by 8)

$-43 = 19y$

$-43/19 = 19y/19$  (divide both sides by 19)

$-43/19 = y$

$y = -43/19$

(k)  $1/(y + 1) = 1/(3y + 2)$

$(y + 1) = (3y + 2)$  (reciprocate both sides)

$y - 3y + 1 = 3y - 3y + 2$  (subtract "3y" from both sides)

$-2y + 1 = 2$

$-2y + 1 - 1 = 2 - 1$  (subtract both sides by 1)

$-2y = 1$

$-2y/-2 = 1/-2$  (divide both sides by -2)

$y = -1/2$

$$(1) \frac{3}{y+1} = \frac{2}{4y+1}$$

$$\frac{y+1}{3} = \frac{4y+1}{2} \text{ (reciprocate both sides)}$$

Eliminate fractions, multiply both sides by 3 and 2

$$3 \times 2 \times \frac{y+1}{3} = 3 \times 2 \times \frac{4y+1}{2}$$

$$2(y+1) = 3(4y+1)$$

$$2y+2 = 12y+3 \text{ (multiply out brackets on both sides)}$$

$$2y-12y+2 = 12y-12y+3 \text{ (subtract "12y" from both sides)}$$

$$-10y+2 = 3$$

$$-10y+2-2 = 3-2 \text{ (subtract both sides by 2)}$$

$$-10y = 1$$

$$-10y/-10 = 1/-10 \text{ (divide both sides by -10)}$$

$$y = -1/10$$