## Chapter

## Learning curve theory

### 2.1 Learning or experience curve theory

Learning curve theory is the theory that as output doubles the average time per unit (total time for all units divided by the units produced) drops by a fixed percentage each time this happens. The same theory can be applied to find the average cost per unit as well as the average time.

## Assumptions

- Heavy manual element in production and so scope for learning.
- Repetitive work so therefore speed and accuracy can be improved.
- Production is in its early stages and so no great improvements have been made.
- Consistency in the workforce means that the same workers are used.
- No extensive breaks in production so that the skills and techniques learned are retained.

Learning curve theory can be illustrated using the following data:
If one unit takes 10 hours to produce then

- Would 2 units take 20 hours to make?
- Would 4 units take 40 hours to make?

If labour can learn or gain by experience, then the answer should be 'no' and average labour hours per unit would fall.

If it is found that an $80 \%$ learning curve applies, it means that each time output doubles on a cumulative basis, the cumulative average labour hours per unit fall by $20 \%$

Thus

|  |  | Average <br> hours/unit |
| :--- | :--- | :---: |
| 1 unit takes | 10 hours | 10 |
| 2 units takes | $(2 \times 10) \times 80 \%=16 \mathrm{hrs}$ | 8 |
| 4 units takes | $(4 \times 10) \times 80 \% \times 80 \%=25.6 \mathrm{hrs}$ | 6.4 |
| 8 units takes | $(8 \times 10) \times 80 \% \times 80 \% \times 80 \%=40.96 \mathrm{hrs}$ | 5.12 |

Note that the average hours per unit falls to $80 \%$ of its previous level each time cumulative output doubles.

This could only occur where labour can learn and therefore influence the time taken; it clearly could not apply where the rate of production is machine or process controlled.

The learning curve can be plotted on normal graph paper.


## Application of learning curve theory

Product pricing - in a competitive situation, pricing on long run average costs can consider the extent to which costs will fall because of the learning curve effect.
Setting standards for cost centres - if labour performance is to be judged against a standard cost, the standard should reflect anticipated learning benefits.
Budgeting for manpower needs - if learning is not considered, over manning could result when new working practices are being introduced.

## Formulae

$Y=a X^{b}$
$\mathbf{Y}=$ average time for that (X) number of units or the average cost per unit
$\mathbf{a}=$ time for the first unit or the cost for the first unit
$\mathbf{X}=$ the number of units you want to calculate an average time or cost for
$\mathbf{b}=$ the index of learning $(\log \mathrm{r} / \log 2)$
If the learning rate of the workforce was $90 \%(r=0.9)$ then the average time or cost per unit falls by $10 \%$ each time output doubles.

Learning curves are relevant when the process is labour intensive or new product therefore new processes are being introduced for the first time. It is likely to affect variable overhead as well as your labour cost. The experience curve is the same principle however is more relevant to falling cost, due to technological, organisational changes and management experience or knowledge gained over many years rather than just learning new skills.

The learning curve formulae can be developed to calculate the total time or total cost for the cumulative $X$ units by adding +1 to the power $b$ as follows:

Total Time $($ or cost $)$ for $X$ units $=\mathrm{aX}^{\mathrm{b}+1}$

## Example 2.1

Locus cars are a company set up in London to build mass-produced sports cars. 5 skilled workmen working a standard 35 -hour week will build these cars. Production has only just started but the workforces are keen to learn quickly.

The first prototype was built however; the time taken had not been recorded. The third unit however did record that the average time for all units at this point was 11.36 hours. Based on previous knowledge, management believe there is a $70 \%$ learning rate of the new workmen.
a) Calculate the time taken for the first prototype?
b) Calculate the total time taken for the $12^{\text {th }}$ car only?
c) Calculate the total time for the $14^{\text {th }}$ to the $15^{\text {th }}$ car only?
d) How many cars can Locus build in the first 2 months assuming the learning effect ceases at 200 cars?
e) Locus cars have also built tractors in the past, and they had built 8 of them. The first tractor took 17 hrs of labour time and the total time for the first $\mathbf{8}$ tractors was $\mathbf{1 0 0}$ hrs. Calculate the learning curve to the nearest $\mathbf{1 \%}$.

### 2.2 Pareto analysis

'Vilfredo Pareto (an Italian economist) once stated that $20 \%$ of the population of Italy at that time accounted for $80 \%$ of the countries wealth'

Such analysis does not always give an exact $20 / 80$ relationship however is still useful to determine what is important.
$20 \%$ of stock lines could be up to $80 \%$ of the value of total stock.
$20 \%$ of customers could account for $80 \%$ of your sales and the same $20 \%$ may or may not account for $80 \%$ of your profit or contribution..
$20 \%$ of your cost drivers could account for $80 \%$ of your fixed overhead when using an ABC system.

All the above examples help understand your position better. A Pareto analysis applied to stock above could mean that if a company did not worry too much about the immaterial components or raw material value, it could consume less overhead in doing so and manage stock more efficiently. In the case of sales above, more client care could be devoted to those customers generating the most income or profit for the company.

## Example 2.2 - Worked example

The monthly sales of a company's four main product groups has been analysed as given below. It is desired to know whether it conforms to a Pareto distribution.

| Product group | Monthly Sales ( $\left.\mathbf{£}^{\prime} \mathbf{0 0 0}\right)$ |
| :--- | :---: |
| Food | 228 |
| Cosmetics | 15 |
| Clothing | 942 |
| Furnishings | 315 |
|  | $-1,500$ |
|  | - |

Solution

- Note: product groups should be accumulated in descending order of value

| Product Group | Sales $\left(\boldsymbol{£}^{\prime} \mathbf{0 0 0}\right)$ | \% | Cum. \% |
| :--- | :---: | :---: | :---: |
| Clothing | 942 | 62.8 | 62.8 |
| Furnishings | 315 | 21.0 | 83.8 |
| Food | 228 | 15.2 | 99.0 |
| Cosmetics | 15 | 1.0 | 100.0 |
|  | $\underline{1,500}$ |  |  |
|  |  |  |  |

Pareto diagram


An alternative presentation could also have been to represent the above as a Pareto curve.

## Example 2.3

Pet rescue centre has been raising funds by asking donators to sponsor an animal that may never have a loving home. Details about the number of sponsorships received over the last 4 months are as follows;

|  | $£$ |
| :--- | ---: |
| Reptiles | 4,000 |
| Puppies | 15,000 |
| Apes | 1,000 |
| Kittens | 3,000 |
| Hamsters | 600 |
| Rats | 200 |
| Fish | 1,000 |
| Chinchillas | $\underline{100}$ |
| Total | $\underline{24,900}$ |

Conduct a Pareto analysis on the above information, to decide the most popular animals that they should use in order to generate more income?

## Key summary of chapter

Learning curve theory - output doubles the average time per unit drops by a fixed percentage.
$\mathrm{Y}=\mathrm{a} \mathrm{X}^{\mathrm{b}}$
$\mathbf{Y}=$ average time for that (X) number of units or the average cost per unit
$\mathbf{a}=$ time for the first unit or the cost for the first unit
$\mathbf{X}=$ the number of units you want to calculate an average time or cost for
$\mathbf{b}=$ the index of learning $(\log \mathrm{r} / \log 2)$
Total Time (or cost) for $X$ units $=a X^{b+1}$

Pareto analysis - helps in determining what is important for example 20\% of a data set could account for $80 \%$ of the activity.

# Solutions to lecture examples 

## Example 2.1

## Calculate the time taken for the first prototype?

$\log 0.7 / \log 2=-0.5146$
11.36 hours $=\mathrm{a} \times 3$ (to the power of -0.5146 )
$11.36=\mathrm{ax} 0.5682$
$\mathrm{a}=11.36 / 0.5682$
$a=20$ hours

## Calculate the total time taken for the $12^{\text {th }}$ car only?

Time for the first 12 units ( $20 \times 12$ (to the power of -0.5146 ) $\times 12=66.81$ hours Time for the first 11 units ( $20 \times 11$ (to the power of -0.5146 ) $\times 11=(\underline{64.05})$ hours Time for the $12^{\text {th }}$ unit 2.76 hours

Calculate the total time for the $14^{\text {th }}$ to the $15^{\text {th }}$ car only?
Time for the first 15 units ( $20 \times 15$ (to the power of -0.5146 ) $\times 15=74.46$ hours
Time for the first 13 units ( $20 \times 13$ (to the power of -0.5146 ) x $13=(\underline{69.46})$ hours Time for the $14^{\text {th }}$ to $15^{\text {th }}$ unit $\quad 5.00$ hours

How many cars can Locus build in the first 2 months assuming the learning effect ceases at 200 cars?

Time for the first 200 units ( $20 \times 200$ (to the power of -0.5146 ) $\times 200=$
Time for the $200^{\text {th }}$ unit
Workmen work in 2 months ( 8 weeks x 5 men x 35 hrs ) 1,400 hours
The first 200 units take 261.8 hours as above

1,400 hours less 261.8 hours $=1,138.2$ hours remaining
$1,138.2$ hours/ 0.6 hours (time for the $200^{\text {th }}$ unit and beyond) $=1,897$ units
Therefore 200 units $+1,897$ units $=2,097$ cars built in the first 2 months

Locus cars have also built tractors in the past, and they had built 8 of them. The first tractor took $\mathbf{1 7} \mathbf{~ h r s ~ o f ~ l a b o u r ~ t i m e ~ a n d ~ t h e ~ t o t a l ~ t i m e ~ f o r ~ t h e ~ f i r s t ~} \mathbf{8}$ tractors was $\mathbf{1 0 0} \mathbf{~ h r s .}$ Calculate the learning curve to the nearest $1 \%$.
$\mathrm{a}=17 \mathrm{hrs}, \mathrm{y}=100 \mathrm{hrs} / 8$ tractors $=12.5 \mathrm{hrs}$ on average for 8 tractors
If we assume that " r " represents the learning rate, then:

| Tractors | Average hours per unit |
| :--- | :--- |
| 1 | 17 |
| 2 | $17 \times \mathrm{r}$ |
| 4 | $17 \times \mathrm{x} \mathrm{r}$ |
| 8 | $17 \times \mathrm{x} \mathrm{rx} \mathrm{r}$ |

Therefore for 8 tractors:
$17 \mathrm{r}^{3}=12.5$
$\mathrm{r}^{3}=12.5 / 17$
$\mathrm{r}^{3}=0.735294117$
$r=(0.735294117)$ to the power of $1 / 3$
$\mathrm{r}=0.90$
Therefore learning rate is $90 \%$

## Example 2.3

Note: product groups should be accumulated in descending order of value

| Product Group | Sales (£'000) | \% | Cum. \% |
| :--- | :---: | :---: | :---: |
| Puppies | 15,000 | 60.2 | 60.2 |
| Reptiles | 4,000 | 16.1 | 76.3 |
| Kittens | 3,000 | 12.1 | 88.4 |
| Apes | 1,000 | 4.0 | 92.4 |
| Fish | 1,000 | 4.0 | 96.4 |
| Hamsters | 600 | 2.4 | 98.8 |
| Rats | 200 | 0.8 | 99.6 |
| Chinchillas | $\underline{100}$ | $\underline{0.4}$ | 100.0 |
| Total | $\underline{24,900}$ | $\underline{100.0}$ |  |

A Pareto curve or bar chart could be produced with the above information. There exists a $76.3 \% / 25 \%$ relationship between puppies and reptiles indicating that these are the biggest revenue earners in the case of sponsorship revenue received. Pet Rescue should therefore concentrate on these two animals or perhaps kittens as well in order to increase the effectiveness of donations received (if you include kittens there exists an $88.4 \% / 37.5 \%$ relationship above).

